Total No. of Questions—**8**] [Total No. of Printed Pages—**6**

Seat No.	

[5668]-181

S.E. (Computer) (I Sem.) EXAMINATION, 2019

DISCRETE MATHEMATICS

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Neat diagrams must be drawn wherever necessary.

(*ii*) Figures to the right indicate full marks.

(*iii*) Assume suitable data, if necessary.

1. (*a*) Prove that the set of rational numbers is countably infinite. [3]

(*b*) Show that for natural no. n : [3]

$$1^3 + 2^3 + 3^3 + ... + n^3 = (1 + 2 + ... + n)^2 \cdot$$

(*c*) Let

$$X = \{1, 2, ... n\}$$

$$R = \{(x, y) \mid x - y\}$$

is divisible by 3. Show that R is equivalence relation. Draw the diagraph for R where $n = 7$. [6]

Or

2. (*a*) In a survey of 60 people : [3]

25 read newsweek magazine

26 read time

26 read fortune

9 read both newsweek and fortune

11 read both newsweek and time

8 read both time and fortune

8 read no magazine at all.

(*i*) Find the no. of people who read all the three magazines.

(*ii*) Find the no. of people who read exactly one magzine.

(*b*) Let $A = \{\phi, b\}$ construct the following sets :

(*i*) $A - \phi$

(*ii*) $\{\phi\} - A$

(*iii*) $A \cup P(A)$

where P(A) is a power set. [3]

(*c*) Let

$$A = \{1, 2, 3, 4, 5\}$$

Define the following relation R on A aRb if and only if $a < b$.

Find :

(*i*) R in roster form

(*ii*) Domain and range of R

(*iii*) Diagraph of R. [3]

(*d*) Draw Hasse diagram representing the partial ordering :

$$\{(a, b) \mid a \text{ divides } b\} \text{ on } \{1, 2, 3, 4, 6, 8, 12\}.$$

Find *two* examples of chain and antichain. [3]

3. (*a*) The company has 10 members on its board of directors. In how many ways can they elect a president, a vice president, a secretary and a treasurer ? [3]

(*b*) Find 8th term in the expansion of $(x + y)^{13}$. [3]

(*c*) Can a simple graph exist with 15 vertices, each of degree five ? [3]

(*d*) For which values of n, m are the following graph regular : [3]

(*i*) K_n

(*ii*) S_n

(*iii*) $G_{n, m}$.

Or

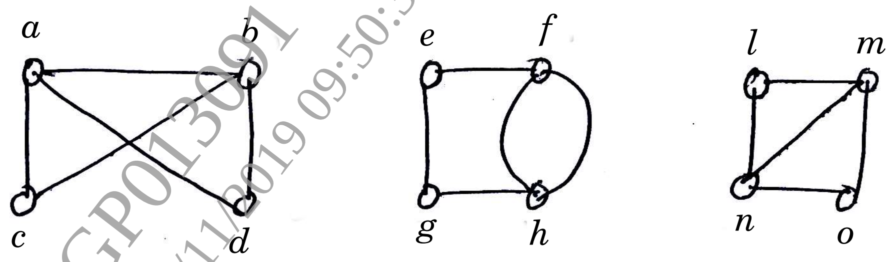
4. (*a*) A box contains 6 white and 5 black balls. Find number of ways 4 balls can be drawn from the box, if :

(*i*) Two must be white

(*ii*) All of them must have same colour. [3]

(*b*) Expand $(3x - 4)^4$ using binomial theorem. [3]

(*c*) Determine whether the following graphs are isomorphic to each other. [3]



[A] [B] [C]

(*d*) How many regions would there be in a plane graph with 10 vertices each of degree 3. [3]

5. (*a*) Construct a binary search tree : [4]

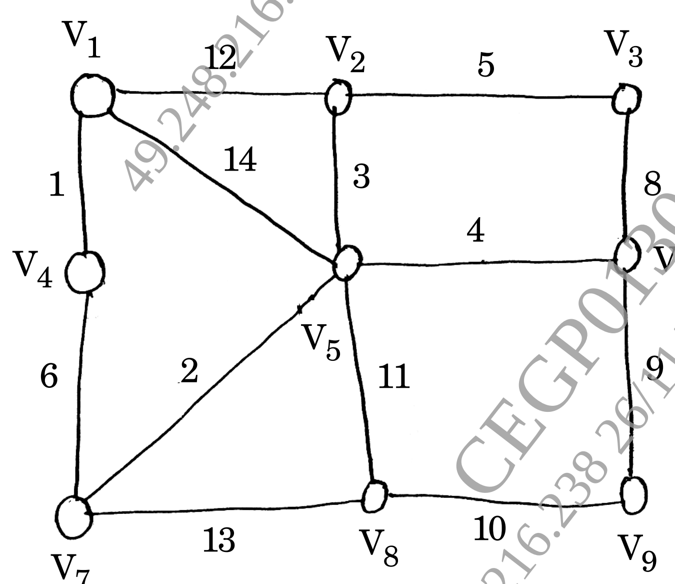
J, R, D, G, W, E, M, H, P, A, F, Q.

(*b*) Construct the binary tree with prefix codes representing : [4]

(*i*) *a* : 11, *e* : 0, *t* : 101, *s* : 100

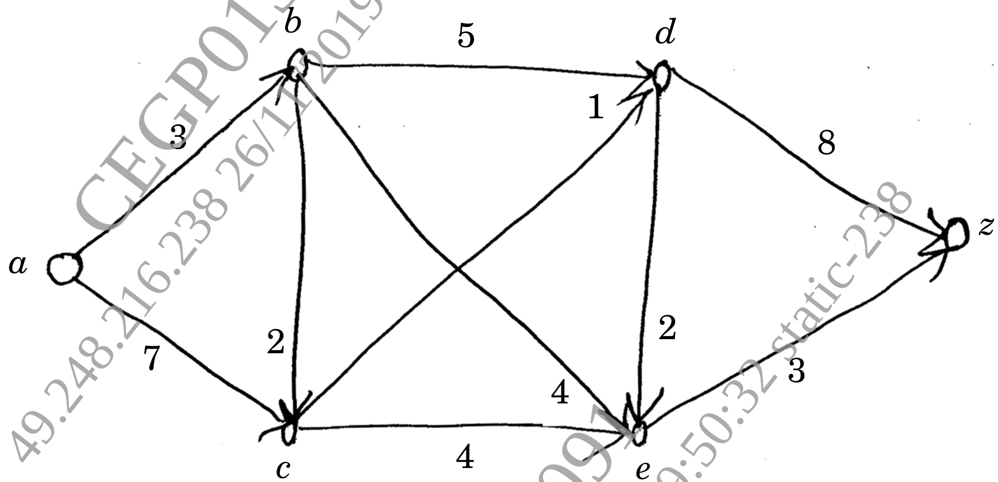
(*ii*) *a* : 1010, *e* : 0, *t* : 11, *s* : 1011, *n* : 1001, *i* : 10001.

(*c*) Give the stepwise construction of minimum spanning tree using Kruskal's algorithm for the following graph. Obtain the total cost of minimum spanning tree. [5]

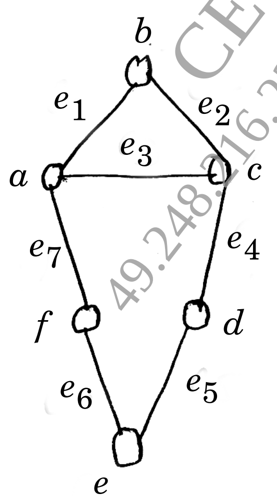


Or

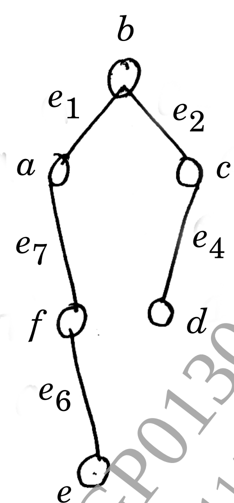
6. (*a*) Using the labelling procedure to find maximum flow in the transport network in the following figure. Determine the corresponding minimum cut. [7]



(*b*) Find fundamental cutsets and circuits of the following graph G_1 with respect to spanning tree T_1. [4]



G_1



T_1

(*c*) Define with example : [2]

Level and height of a tree.

7. (*a*) Write properties of Binary operations. [5]

(*b*) Prove that the set 2 of all integers with binary operation * defined by :

$$a * b = a + b + 1 \quad \forall\, a, b \in 2$$

is an abelian group. [5]

(*c*) Let $A = \{0, 1\}$. Is A closed under : [3]

(1) Multiplication

(2) Addition.

Or

8. (*a*) $f : G \to G$, G is group with identity 'e' such that $f(a) \in e$ for all $a \in G$ prove that function f is homomorphism. [5]

(*b*) In the set R of real number. Decide whether the following composition is associative $a, b, c \in R$: [4]

(1) $a * b = a + 2b$

(2) $a * b = a$.

(*c*) Prove that the set :

$$A = \{0,\ 2,\ 4,\ 6,\ 8\}$$

with t_{10} and x_{10} operation i.e. $R = \{A, t_{10}, x_{10}\}$ is a ring. [4]